Outline of the session

- Why?
- Recommendation
- Possible dataset
  - Experimental dataset
  - Modelled dataset
- Example of wave climate characterisation at the DanWEC test site
- Introduction to extreme wave analysis

- Exercises:
  - Compute a scatter table
  - Determine extreme values from POT method
Wave energy resource assessment... Why?

• Would you choose to place a device here?
Wave energy resource assessment... Why?

- There is a need to understand the wave climate characteristics to properly convert the wave energy resource into useful energy.
Wave energy resource assessment... Why?

- Moon: 100% mapped
- Mars: 100% mapped
- Earth's ocean: 5% mapped
Overview of the global wave energy resource

• The main areas of wave energy resource occur in bands across the Northern and Southern hemispheres, with less energetic regions close to the equator and poles.
Overview of the global wave energy resource

- The main areas of wave energy resource occur in bands across the Northern and Southern hemispheres, with less energetic regions close to the equator and poles.

Annual mean power density is not enough. Other important factors are to be taken into account while considering the specificity of the individual wave energy converters.
Visual inspection

- Prevailing winds on earth

- The waves that reach the western coast of Europe are typically larger than those that reach the eastern coast of the USA because not only do the winds normally blow west to east, but the typical track of the weather systems in the North Atlantic is west to east.
Visual inspection

• Diffraction and interference: Waves that pass through small gaps (such as between the island and the mainland) diffract – they bend outwards. This can also lead to constructive and destructive interference patterns on the far side of the island.
Visual inspection

- Knowledge of the typical wind directions and fetch lengths can be used to provide an initial indication of the type of waves

<table>
<thead>
<tr>
<th>Location</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mediterranean Sea</td>
<td>Small fetch length</td>
</tr>
<tr>
<td>South Pacific Ocean</td>
<td>Large fetch length and relatively high winds</td>
</tr>
<tr>
<td>Equatorial regions</td>
<td>Small wind speeds</td>
</tr>
</tbody>
</table>
Visual inspection

- Seasonal weather variations
Visual inspection

- Seasonal weather variations

- The winds are significantly more consistent in the Southern hemisphere, leading to wave resource much less variable than in the Northern hemisphere.
Characterisation of ocean waves and wave climate

- Temporal characteristic of a wave climate
  - Daily, seasonal, annual variations

- Directional characteristics of a wave climate

- Spectral characteristics of a wave climate
Temporal characteristic of a wave climate

- DanWEC test site
Temporal characteristic of a wave climate

- DanWEC test site

- The more consistent the wave climate is, the more attractive it is. The WEC and power generating plant can remain closest to its optimal operating conditions.
Directional characteristics of a wave climate

- Example of a wave rose

<table>
<thead>
<tr>
<th>Wave Height Range</th>
<th>Color</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0 : 1.0 m</td>
<td>dark blue</td>
</tr>
<tr>
<td>1.0 : 2.0 m</td>
<td>blue</td>
</tr>
<tr>
<td>2.0 : 3.0 m</td>
<td>cyan</td>
</tr>
<tr>
<td>3.0 : 4.0 m</td>
<td>green</td>
</tr>
<tr>
<td>4.0 : 5.0 m</td>
<td>yellow</td>
</tr>
<tr>
<td>5.0 : 6.0 m</td>
<td>red</td>
</tr>
<tr>
<td>&gt; 6.0 m</td>
<td>dark red</td>
</tr>
</tbody>
</table>

- Directionality of wave is relatively important depending on the type of device
Directional characteristics of a wave climate

• Example of a wave rose

• Directionality of wave is relativ
Spectral characteristics of a wave climate

- Frequency dependent response of (many) WECs
- Average directional spectral variance density at a particular test site
Spectral characteristics of a wave climate

- Frequency dependent response of (many) WECs
- Average directional spectral variance density at a particular test site

Is there a match?
Characterisation parameters

• Significant wave height, 1 symbol with 3 different definitions based on 3 methods:
  • Observation
  • Time domain analysis
  • Frequency domain analysis

• In wave energy, the preferred representation is the one derived from frequency analysis

\[ H_{m0} = 4\sqrt{m_0} \]
Characterisation parameters, $H_s$

- Significant wave height, 1 symbol with 3 different definitions based on 3 methods:
  - Observation
  - Time domain analysis
  - Frequency domain analysis

- In wave energy, the preferred representation is the one derived from frequency analysis

$$H_{m0} = 4\sqrt{m_0}$$

In the good old days...

Definition based on the energy in the waves and directly related to the average wave power density.
Characterisation parameters, $T_p$ vs $T_e$

- $T_e$ (energy period) is the variance weighted mean period of the one-dimensional period variance density spectrum

$$T_e \equiv T_{-10} = \frac{m_{-1}}{m_0}$$

- $T_p$ (peak period) The peak period is the inverse of the frequency associated with the maximum value of the wave spectrum

$$T_p = 1/f_p$$
Characterisation parameters, $T_p$ vs $T_e$

- $T_e$ (energy period) is the variance weighted mean period of the one-dimensional period variance density spectrum
  \[ T_e \equiv T_{-10} = \frac{m_{-1}}{m_0} \]

- $T_p$ (peak period) The peak period is the inverse of the frequency associated with the maximum value of the wave spectrum
  \[ T_p = 1/f_p \]

- For a JONSWAP spectrum with a peak enhancement factor, $\gamma = 3.3$, the ratios of the wave periods are
  \[ 1.12 T_e = 1.29 T_z = T_p \]
Characterisation parameters, $T_p$ vs $T_e$

- Wave spectra analysis as a function of rising and falling wind speed.

Figure: Time series of wind speed and wave height variation February 2016 (Buoy III).
Characterisation parameters, $T_p$ vs $T_e$

Selected examples of measured wave spectra for $H_s = 1$ m and $H_s = 2$ m
Characterisation parameters

- The omnidirectional wave power $J$ in deep water is defined as:

$$J = \frac{\rho g^2}{64\pi} H_m^2 T_e$$

- The spectral bandwidth $\varepsilon_0$ (the relative spreading of the energy with wave frequency):

$$\varepsilon_0 = \sqrt{\frac{m_0 m_{-2}}{m^2_{-1}}} - 1$$

- The mean zero-crossing period of the waves $T_z$ can be useful as it can allow a spectrum to be scaled using assumptions regarding the spectral shape:

$$T_z \approx T_{02} = \sqrt{\frac{m_0}{m_2}}$$
Characterisation parameters: directionality

- The directionally resolved wave power density $J(\theta)$ is a key directional characteristic of the sea-state as it defines the wave power propagation in a particular direction.

\[
J(\theta) = \rho g \int_{-\pi}^{+\pi} \int_{0}^{\infty} S(\omega, \phi) C_g(\omega) \cos(\theta - \phi) \delta \cdot d\omega \cdot d\phi
\]

\[
\delta = 1, \quad \cos(\theta - \phi) \geq 0
\]

\[
\delta = 0, \quad \cos(\theta - \phi) < 0
\]

- Direction of maximum directionally resolved wave power $J_{\theta \max}$: $\theta_{\max}$

- The directional spreading of wave power is the directionality coefficient, which is the ratio of the maximum directionally resolved wave power to the omni-directional wave power:

\[
d = \frac{J_{\theta \max}}{J}
\]
Representation of the wave climate

- Scatter diagram representation of the wave climate:
  - A frequency of occurrence table indexed by a representative wave period (energy period) and a representative wave height

<table>
<thead>
<tr>
<th>Significant Wave Height (m)</th>
<th>Energy Period (s)</th>
<th>&lt;5</th>
<th>5-6</th>
<th>6-7</th>
<th>7-8</th>
<th>8-9</th>
<th>9-10</th>
<th>10-11</th>
<th>11-12</th>
<th>12-13</th>
<th>13-14</th>
<th>&gt;14</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt;5</td>
<td></td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.01</td>
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<tr>
<td>4,5-5</td>
<td></td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<tr>
<td>4-4,5</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.01</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<tr>
<td>3,5-4</td>
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<td>0.00</td>
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<td>0.01</td>
<td>0.04</td>
<td>0.03</td>
<td>0.00</td>
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<td>0.00</td>
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<td>0.00</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.43</td>
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<tr>
<td>2,5-3</td>
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<td>0.00</td>
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<td>0.36</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.76</td>
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<tr>
<td>2-2,5</td>
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<td>0.00</td>
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<td>1.76</td>
<td>2.03</td>
<td>0.80</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<td>0.00</td>
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<tr>
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<td>0.01</td>
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<td>1.82</td>
<td>12.86</td>
<td>16.34</td>
<td>6.89</td>
<td>3.18</td>
<td>0.08</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>41.28</td>
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<td>0.09</td>
<td>0.46</td>
<td>3.61</td>
<td>2.37</td>
<td>2.50</td>
<td>2.65</td>
<td>0.05</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>11.74</td>
</tr>
<tr>
<td>%</td>
<td></td>
<td>0.25</td>
<td>2.61</td>
<td>23.81</td>
<td>37.81</td>
<td>24.85</td>
<td>10.45</td>
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<td>0.02</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>100.00</td>
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</table>
Recommendation for a wave energy resource assessment and characterization

- IEC 62600-101 TS recommends:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Class of Assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean water depth</td>
<td>m</td>
<td>• Required</td>
</tr>
<tr>
<td>Annual mean omni-directional wave power</td>
<td>kW/m</td>
<td>• Required</td>
</tr>
<tr>
<td>Extent of successful model validation</td>
<td></td>
<td>• Required</td>
</tr>
<tr>
<td>Monthly variability of omni-directional wave power</td>
<td>kW/m</td>
<td>○ Recommended</td>
</tr>
<tr>
<td>Annual mean significant wave height</td>
<td>m</td>
<td>○ Required</td>
</tr>
<tr>
<td>Monthly variability of significant wave height</td>
<td>m</td>
<td>○ Required</td>
</tr>
<tr>
<td>Annual mean energy period</td>
<td>s</td>
<td>○ Required</td>
</tr>
</tbody>
</table>
Recommendation for a wave energy resource assessment and characterization

- IEC 62600-101 TS recommends:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Class of Assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly variability of energy period</td>
<td>s</td>
<td>○</td>
</tr>
<tr>
<td>Annual mean spectral width</td>
<td>-</td>
<td>○</td>
</tr>
<tr>
<td>Monthly variability of spectral width</td>
<td>-</td>
<td>○</td>
</tr>
<tr>
<td>Annual mean of maximum directionally resolved wave power</td>
<td>kW/m</td>
<td>○</td>
</tr>
<tr>
<td>Monthly variability of maximum directionally resolved wave power</td>
<td>kW/m</td>
<td>○</td>
</tr>
<tr>
<td>Annual mean direction of maximum directionally resolved wave power</td>
<td>deg</td>
<td>○</td>
</tr>
<tr>
<td>Monthly variability of direction of maximum directionally resolved wave power</td>
<td>deg</td>
<td>○</td>
</tr>
<tr>
<td>Annual mean directionality coefficient</td>
<td>-</td>
<td>○</td>
</tr>
<tr>
<td>Monthly variability of directionality coefficient</td>
<td>-</td>
<td>○</td>
</tr>
</tbody>
</table>
Recommendation for a wave energy resource assessment and characterization

- IEC 62600-101 TS recommends:
  - Annual, seasonal and monthly scatter table
  - Wave rose
  - Visual representation of the distribution of important parameters for different months and years
Representation of the wave climate

• Practical example:
  o Exercise 1: Scatter diagram representation of the wave climate
Challenges in Wave Climate Characterisation

• A wave climate can be reasonably approximated as a long-term series of sea-states that are defined by the directional wave spectrum together with water depth, marine current speed/direction and wind speed/direction (10 years)
Challenges in Wave Climate Characterisation

• A wave climate can be reasonably approximated as a long-term series of sea-states that are defined by the directional wave spectrum together with water depth, marine current speed/direction and wind speed/direction (10 years)

• Data not available at that level of details
• Too much data
Challenges in Wave Climate Characterisation

- A wave climate can be reasonably approximated as a long-term series of sea-states that are defined by the directional wave spectrum together with water depth, marine current speed/direction and wind speed/direction (10 years)

- Data not available at that level of details
- Too much data

A characterisation of the wave climate at a single point or over an area is used.
Challenges in Wave Climate Characterisation

- The average omni-directional wave power is probably the most common characterisation of the wave resource for the assessment of wave energy.
- The areas with higher average omni-directional wave power, such as the north-west coast of Europe, are also the areas with the most interest in the deployment of wave energy converters.
Challenges in Wave Climate Characterisation

- The average omni-directional wave power is probably the most common characterisation of the wave resource for the assessment of wave energy.
- The areas with higher average omni-directional wave power, such as the north-west coast of Europe, are also the areas with the most interest in the deployment of wave energy converters.

Assumption is that a wave energy converter’s power capture is proportional to the average omni-directional wave power thus a larger average omni-directional wave power equates to a larger power capture.
Challenges in Wave Climate Characterisation

- Six main processes responsible for the change in average omni-directional wave power as waves propagate
  - Shoaling
  - Refraction
  - Diffraction
  - Depth-induced wave breaking
  - Bottom friction
  - Wind growth
Challenges in Wave Climate Characterisation

- Average incident wave power at the European Marine Energy Centre, Orkney, Scotland

- Gross power: omni-directional wave power

- Net power: includes the effect of wave refraction

- Exploitable power: includes the effect of refraction, wave breaking and bottom friction
Types of datasets for wave climate characterisation

- Experimental data from actual measurement of the ocean waves

- Modelled data from numerical modelling of ocean waves

\[
\frac{\partial N}{\partial t} + \nabla_x \cdot \left[ C_g + U \right] N + \frac{\partial C_\sigma N}{\partial \sigma} + \frac{\partial C_\theta N}{\partial \theta} = \frac{S_{tot}}{\sigma}
\]
Measurement of Ocean Waves

- Surface-Following Buoy
  - buoy slackly moored to the seabed
  - vertical motion is typically measured using an accelerometer
  - double integrated to provide a time-series of the water surface elevation
  - recorded surface elevation is used to estimate the wave spectrum and from that the sea-state parameters
  - may also contain instruments to measure the inclination of the buoy so that the direction of the waves
  - accuracy is well established
Measurement of Ocean Waves

- Surface-Following Buoy
  - strong currents and steep waves both reduce the accuracy of the measurements because the buoy does not exactly follow the water surface
  - relatively expensive
  - relatively high risk of loss of the instrument
Measurement of Ocean Waves

- **Sea-Bed Pressure Sensor**
  - Cheaper alternative to the surface-following buoy
  - measure the variation in pressure and from that infer the water surface elevation
  - attenuation of wave pressure with depth means that they are only suitable for relatively shallow water
  - also means that high frequency wave are attenuated
  - depth limit of about 10–20 m is typical
  - sea-bed pressure sensors may be deployed in an array to provide information on the directional distribution of the waves
Measurement of Ocean Waves

• **Acoustic Current Profiler**
  • measures the water velocity using the red/blue shift in acoustic pulses for the instrument
  • water velocities as determined from each beam are combined and processed to produce a time-series of the 3D wave-induced water velocities and from that the directional wave spectrum.
  • deployed on the seabed where they are less susceptible to damage and bio-fouling
  • suitable for water depth of less than about 50 m
  • gives further insight on local marine currents
  • because located on the sea-bed, the data is typically stored on board.
Measurement of Ocean Waves

- Land-based radar
  - remote sensor to measure the waves avoiding deployment in an aggressive environment
  - data readily available
  - typically lost cost
  - radar provides information on the sea-state by analysing the backscatter from the waves over an area
  - coupled analysis from multiple locations means that the directional wave spectrum can often be calculated, together with sea-state parameters

Land-based radar system on the dike at Petten (NL)
Measurement of Ocean Waves

- Radar altimetry
  - radar altimeters are available in a number of satellites that are circling the earth
  - provide estimates of the $H_s$ along the track of the satellite
  - low temporal resolution of the wave resource data as a satellite may only pass over a point every 10–35 days
  - provide wave resource data over a large geographical area
  - current developments exist to use satellite data to produce estimates of the wave period
Modelling of Ocean Waves

• Why?
  o It is unlikely that data from a wave measuring instrument for the desired location will exist.
  o Knowledge of the average wave climate requires many years of data and it is not typically possible to deploy a wave measuring instrument for the time required to produce the required information.
Modelling of Ocean Waves

• Why?
  o It is unlikely that data from a wave measuring instrument for the desired location will exist.
  o Knowledge of the average wave climate requires many years of data and it is not typically possible to deploy a wave measuring instrument for the time required to produce the required information

➢ Third generation spectral wave models
  ▪ SWAN (Open source)
  ▪ TOMAWAC (Open source)
  ▪ Mike21 SW
Modelling of Ocean Waves

• The principle of third generation spectral wave models is to solve the action balance equation for action density $N$

$$\frac{\partial}{\partial t} N(\sigma, \theta) + \frac{\partial}{\partial x} c_x N(\sigma, \theta) + \frac{\partial}{\partial y} c_y N(\sigma, \theta) + \frac{\partial}{\partial \sigma} c_\sigma N(\sigma, \theta) + \frac{\partial}{\partial \theta} c_\theta N(\sigma, \theta) = \frac{S(\sigma, \theta)}{\sigma}$$

• Where
  • action density $N$ is the energy density divided by the angular frequency relative to any currents present, $\sigma$
  • $c_\sigma$ and $c_\theta$, the propagation velocities in the $\sigma$- and $\theta$- space
  • $c_x$ and $c_y$ are the velocity components of $N(\sigma, \theta)$
  • $S(\sigma, \theta)$, the source term representing the generation, redistribution and dissipation of energy in the spectrum
Modelling of Ocean Waves

- In third generation spectral wave models in third generation spectral wave models
Modelling of Ocean Waves

• In third generation spectral wave models in third generation spectral wave models

- The wind input source term represents the energy that is transferred from the wind into the waves
Modelling of Ocean Waves

- In third generation spectral wave models in third generation spectral wave models

The quadruplet wave-wave interactions are associated with multiple resonant couplings between sets of four wave components that cause energy transfer via non-linear interactions.
Modelling of Ocean Waves

• In third generation spectral wave models in third generation spectral wave models

- The quadruplet wave-wave interactions are associated with multiple resonant couplings between sets of four wave components that cause energy transfer via non-linear interactions.
Modelling of Ocean Waves

- In third generation spectral wave models, when the steepness of a wave becomes too large, the top of the wave becomes unstable and the wave breaks resulting in white-capping.
Modelling of Ocean Waves

- In third generation spectral wave models, the bottom friction source term represents the energy transfer from the waves to turbulence induced by shear stress from fluid flow over the bottom.
Modelling of Ocean Waves

- In third generation spectral wave models in third generation spectral wave models

- Triad wave-wave interactions are associated with multiple resonant couplings between wave components, but in this case they are between sets of three wave components that cause energy transfer via non-linear interactions. They only occur in very shallow water.
Modelling of Ocean Waves

- In third generation spectral wave models, depth-induced wave breaking source term typically assumes that a fixed proportion of the energy in any wave is lost when it breaks.
Modelling of Ocean Waves

- Grid definition
  - resolution of the grid will depend on the desired accuracy of the model, with higher resolution grids typically resulting in more accurate models, but at the expense of greater computational effort
Modelling of Ocean Waves

• Some uncertainties
  • uncertainty in the grid design
  • uncertainties in the model boundary data (e.g. the waves on the model boundary, the bathymetry, the variation in the seabed bottom friction coefficient, the choice of source term models)
Modelling of Ocean Waves

- Some uncertainties
  - uncertainty in the grid design
  - uncertainties in the model boundary data (e.g. the waves on the model boundary, the bathymetry, the variation in the seabed bottom friction coefficient, the choice of source term models)

> Any numerical model should be validated using a wide range of different conditions that include what may be expected over the whole year.
DanWEC wave climate characterisation

- Numerical spectral wave model using MIKE 21 SW
  - Period 1981-2015
  - Inputs:
    - Wind data from Climate forecast system reanalysis
    - Water level conditions from DHI hydrodynamic model
    - Spectral wave boundary data from DHI Spectral Wave model
DanWEC wave climate characterisation

• Numerical spectral wave model using MIKE 21 SW

Validation of hydrodynamic model and DHI Spectral Wave model from different stations
DanWEC wave climate characterisation

- Numerical spectral wave model using MIKE 21 SW

Bathymetry and final mesh resolution for the local spectral wave model

Model domain for the local spectral wave model
DanWEC wave climate characterisation

- Numerical spectral wave model using MIKE 21 SW

Zoom of the final mesh around DanWEC test site

Model domain for the local spectral wave model
DanWEC wave climate characterisation

- Numerical spectral wave model using MIKE 21 SW

Validation of the model with measured data from surface-following buoys

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Harbour buoy - Buoy I</td>
<td>DWR</td>
<td>8.5821</td>
<td>57.1315</td>
<td>-17.5</td>
<td>01-12-2005</td>
<td>24-01-2016</td>
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<tr>
<td>Buoy NE - Buoy II</td>
<td>DWR</td>
<td>8.6192</td>
<td>57.1536</td>
<td>-27.5</td>
<td>28-02-2015</td>
<td>13-04-2016</td>
</tr>
</tbody>
</table>
DanWEC wave climate characterisation

- Numerical spectral wave model using MIKE 21 SW

Validation of the model with measured data from surface-following buoys
Extreme wave analysis

• Significant wave height is a random variable which varies with respect to time and location
Extreme wave analysis

- Significant wave height is a random variable which varies with respect to time and location
- How to determine the design wave height?
Extreme wave analysis

- Return period and encounter probability

The design level is represented by **return period** or **encounter probability**

Some notation:
- $X$: random variable ($H_s$)
- $x$: realisation of $X$
- $F(x)$: Cumulative distribution function of $X$, $F(X) = \text{Prob}(X \leq x)$
- $t$: Number of years of observation of $X$
- $n$: Number of observation in a period $t$
- $\lambda$: Sample intensity, $\lambda = \frac{n}{t}$
Extreme wave analysis

- The probability that an observed $H_s$ will be larger than $x$ is $(1-F(x))$
Extreme wave analysis

- The probability that an observed $H_s$ will be larger than $x$ is $(1-F(x))$
- If the total number of observations is $n$, the number of observations where $(X > x)$ is:

$$k = n \left( 1 - F(x) \right) = t \lambda \left( 1 - F(x) \right)$$
Extreme wave analysis

- The probability that an observed $H_s$ will be larger than $x$ is $(1-F(x))$
- If the total number of observations is $n$, the number of observations where $(X > x)$ is:
  \[ k = n (1 - F(x)) = t \lambda (1 - F(x)) \]
- The return period $T$ of $x$ is defined as
  \[ T = t \bigg|_{k=1} = \frac{1}{\lambda (1 - F(x))} \]
Extreme wave analysis

• Encounter probability $p$

Based on the fact that on average $x$ will be exceeded once in every $T$ years, the exceedence probability of $x$ in 1 year is $1/T$. In the same manner:

- non-exceedence probability of $x$ in 1 year: $\text{Prob}(X \leq x) = 1 - \frac{1}{T}$

- non-exceedence probability of $x$ in 2 years: $\text{Prob}(X \leq x) = \left(1 - \frac{1}{T}\right)^2$

- non-exceedence probability of $x$ in $L$ years: $\text{Prob}(X \leq x) = \left(1 - \frac{1}{T}\right)^L$
Extreme wave analysis

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Based on the fact that on average $x$ will be exceeded once in every $T$ years, the exceedence probability of $x$ in 1 year is $1/T$. In the same manner:

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• non-exceedence probability of $x$ in $L$ years: $\text{Prob}(X \leq x) = \left(1 - \frac{1}{T}\right)^L$

The encounter probability, i.e. the exceedence probability of $x$ within a structure lifetime of $L$ years is

$$p = 1 - \left(1 - \frac{1}{T}\right)^L$$
Extreme wave analysis

- Data sets
  - **Complete data set**: containing all the direct measurements of wave height usually equally spaced in time.
  - **Annual series data set**: consisting of the largest wave height in each year of measurements/hindcasts
  - **Partial series data sets**: composed of the largest wave height in each individual storm exceeding a certain level (threshold). The threshold is determined based on the structure location and engineering experience. It is also called POT data set (Peak Over Threshold).
Extreme wave analysis

- Extreme data sets should fulfill the following 3 conditions:
  - **Independence**: There must be *no correlation between extreme data*. The annual series data set and the partial series data set meet the independence requirement because the extreme data are from different storms.
  - **Homogeneity**: The extreme data must belong to the same statistical population, e.g. *all extreme data are from wind-generated waves*.
  - **Stationary**: There must be *stationary long-term climatology*. Studies of wave data for the North Sea from the last 20 years give evidence of non-stationarity as they indicate a trend in the means. Average variations exist from decades to decades or even longer period of time. However, until more progress is available in investigating long-term climatological variations, the assumption of stationary statistics might be considered realistic for engineering purpose, because the long-term climatological variation is generally very weak.
Extreme wave analysis

- Candidate distributions:
  - Exponential
    \[ F = F_X(x) = P(X < x) = 1 - e^{-\left(\frac{x-B}{A}\right)} \]
  - Weibull
    \[ F = F_X(x) = P(X < x) = 1 - e^{-\left(\frac{x-B}{A}\right)^k} \]
  - Gumbel
    \[ F = F_X(x) = P(X < x) = e^{-e^{-\left(\frac{x-B}{A}\right)}} \]
  - Frechet
    \[ F = F_X(x) = P(X < x) = e^{-\left(\frac{x}{A}\right)^k} \]
  - Log-normal
    \[ F = F_X(x) = P(X < x) = \Phi\left(\frac{\ln(x) - B}{A}\right) \]

\( A, B, k \): distribution parameters
\( \Phi \): standard normal distribution function
Extreme wave analysis

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\( A, B, k \): distribution parameters
\( \Phi \): standard normal distribution function
Extreme wave analysis

- Design wave height: $x^T$
  - Distribution rewritten in terms of the realisation of a wave height:
    - **Weibull** $x = A \left( -\ln(1 - F) \right)^{\frac{1}{k}} + B$
    - **Gumbel** $x = A \left( -\ln(-\ln(F)) \right) + B$

$A$, $B$, $k$: distribution parameters
Extreme wave analysis

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  $$\lambda = \frac{\text{number of extreme data}}{\text{number of years of observation}}$$

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Extreme wave analysis

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  \[ A, B, k: \] distribution parameters

• Define the sample intensity

\[ \lambda = \frac{\text{number of extreme data}}{\text{number of years of observation}} \]

• Return period

\[ T = \frac{1}{\lambda \left( 1 - F \right)} \quad \text{or} \quad F = 1 - \frac{1}{\lambda T} \]
Extreme wave analysis

• Design wave height: $x^T$

• Distribution rewritten in terms of the design wave height:

  - **Weibull**
    $$x^T = A \left( -\ln\left( \frac{1}{x^T} \right) \right)^{\frac{1}{k}} + B$$

  - **Gumbel**
    $$x^T = A \left( -\ln\left( -\ln\left( 1 - \frac{1}{x^T} \right) \right) \right) + B$$

  $A, B, k$: distribution parameters
Extreme wave analysis

• Practical example:
  o Exercise 2: Extreme wave analysis